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I. Main tasks of the research institute in 2009

The Alfréd Rényi Institute of Mathematics is an important centre of the international mathematical life. In 2009, it preserved its earlier achieved rank and position at the forefront of the world's mathematical research.

The activity of the institute is carried out in nine scientific departments. Because of recent death and retirement, the separate Department of Information Theory was merged into the Department of Probability and Statistics. With the support of Momentum project, the Research Group of Cryptography was established within the Department of Discrete Mathematics. Each department is in close connection with the other centres of its research subject. Due to this fact, they follow their research program on the most recent questions raised in the developments of mathematics.

One research fellow of the institute received the title "Doctor of Academy" in 2009. By the end of the year, 13 members of academy (11 according to time sheet statitistics), 28 doctors of academy (23 stat.) and 26 PhD or candidate degree holders (25 stat.) worked in the institute. There was a particular stress to involve young talents – PhD students or young postdocs – in the research of the institute. In 2009, six additional such young researchers were employed on the four new positions of the Academy, and on the ones becoming vacant. The researchers of the institute supervised 10 PhD students in the joint Doctoral Program with the Central European University. In 2009 altogether, counting the recently appointed young researchers, the institute had the opportunity to train 17 promising talents.

In similar number than in recent years, the fellows of the institute published 166 works, 163 of which are scientific, the other three are popular scientific. The scientific works include three edited volumes, one thesis of academy doctor title, seven PhD thesises, four monographies, three book chapters, 20 publications in conference proceedings, and 125 publications in refereed international journals. Of these 125 papers, 114 appeared in foreign journals, the other 11 in English language international journals published in Hungary.

II. Outstanding research and other results, and their socio-economic impact in 2009

Department of Algebra

- Higman's conjecture about the number of conjugacy classes in the group of unitriangular matrices was investigated. It was found that a generalization of the conjecture to arbitrary pattern subgroups does not hold, which may indicate that the original conjecture also fails for large dimensions.
- It was shown that the generating graph of any sufficiently large finite simple group contains a Hamiltonian cycle.
- Solving a conjecture from 1966 it was shown how large the arithmetic average of the dimensions of fixed point spaces of all elements of a group G can be. Several applications of this result were also given.
- It was shown that the growth of the number of generators in an infinite chain of subgroups of finite index in a finitely presented amenable group is sublinear with respect to the index of the subgroups.
- They showed that if a covering tower of regular graphs is an expander family then either every sufficiently large element of the tower is a bipartite graph or there is a uniform bound which separates the elements of the tower from being bipartite.
- A dense open embedding was found from the moduli space parametrizing representations of an arbitrary quiver into a moduli space belonging to a bipartite quiver. The latter moduli space is a projective algebraic variety.
- A characterization of extended Dynkin quivers among all quivers was found by means of smoothness of all their moduli spaces.
- By means of a recent general invariant theoretic result, the minimal system of relations of generators of the algebra of multisymmetric polynomials was determined in the three-dimensional case.
- Lie algebras can be considered as special Leibniz algebras, therefore it is worth investigating connections between Lie and Leibniz cohomologies and deformations. Well-applicable sufficient conditions were found for these two cohomologies to coincide, and non-Lie deformations were explicitly characterized in two examples.
- By means of a surprising matrix construction, a characterization was given for those inverse monoids which occur as monoids of bi-congruences of finite minimal algebras generating an arithmetical variety.
- Several Morita-equivalent properties were found for semigroups with local identities.
- The TéT cooperation with South African, Bulgarian and Estonian mathematicians has been very successful, resulting in several joint papers in preparation. The cooperation with the Vietnamese Academy is also promising.

Department of Algebraic Geometry and Differential Topology

- Those Seifert fibered 3-manifolds have been classified which carry tight contact structures.
- An invariant of Legendrian and transverse knots in contact 3-manifolds have been defined as an element of the knot Floer homology. Using this invariant the existence of many new phenomena have been verified.
- A combinatorial definition of the (stable version of the) simplest version of Heegaard Floer homology has been given.
- Using symplectic topology those weighted homogeneous singularities have been classified, which admit rational homology disk smoothings.

- They proved that the size of the powers of any system of generators of a Lie type finite simple group is growing exponentially as long as they attain the size of the group. The result has several consequences in number theory and graph theory.
- They proved that the Helly-dimension of any algebraic group is finite.
- They provided surgery formulae for the Seiberg-Witten invariants of knots of complex algebraic singularities. One of the key application is the proof of the Seiberg-Witten Invariant Conjecture for splice-quotient singularities.
- They formulated a generalization of the Monodromy Conjecture of Denef-Loeser, and they verified it for complex plane curve singularities.
- Disproving and old conjecture, they have proved that the signature of complex hypersurface singularities is not semi-continuous under deformation.
- They introduced two exact sequences for the lattice cohomology, which will be crucial in their direct computations and applications.
- The geometry group used efficiently three TéT-Grants (Austrian, French and Spanish).

Department of Algebraic Logic

- They gave finitely many equations that use so-called positive operations and the identity constant, are valid among binary relations, and all such equations can be deduced from them.
- In investigating the theory AccRel which is the theory of special relativity enriched with accelerating observers. Using just the axioms of this theory, they proved that the world-line of a uniformly accelerating observer is a hyperbola.

Department of Analysis

- The order of approximation of convex surfaces by multidimensional convex polynomial was investigated and precisely determined in some cases.
- They defined a new weighted generalization of the classical Kantorovich operator which allows more freedom for the parameters of the Jacobi weight, and proved corresponding approximation properties. They investigated the conditions on the parameters of the Jacobi weight for the convergence of weighted Hermite-Fejér interpolation.
- They have investigated non-periodic tilings of cyclic groups. A general algorithm was presented to find all such tilings in the case when the size of the group is a product of at most two different prime factors. In particular, they have characterized all such tilings of the cyclic group of 144 elements. An interesting motivation of this problem is that these tilings – the so called Vuza canons – are sometimes used by contemporary composers in music pieces.
- They have investigated the MUB-6 problem of quantum-information theory, i.e., the problem of whether there exist 7 mutually unbiased bases in complex dimension 6.
- They have proved that already 4 such bases do not exist in the special case when one assumes that the standard basis and a generalized Fourier basis are present.

Department of Discrete Mathematics

- They continued their research in defining, investigating and applying graph limits and proved important results in the problems as follows.
  - limit objects of graph sequences for dense graph sequences;
  - graph limits for sparse graphs, property testing of sparse graph sequences;
  - applying methods of mathematical logic (like ultraproducts) to prove various versions of Regularity Lemma;
  - proof of the existence of a constant-time algorithm that computes the matching number of a graph.
- They showed significant theorems in extremal graph, hypergraph, and matrix theory:
  - Ramsey type theorems by means of the Regularity Lemma;
  - extremal results for hypergraphs not containing Berge cycles;
  - application of extremal graph and hypergraph results in number theory;
  - eigenvalue estimates for graph sequences with excluded subgraphs, sharpening extremal graph results, using eigenvalue technique;
  - a longstanding conjecture about the density of a subgraph of a high dimensional hypercube not containing any cycle of given length;
  - embedding trees into graphs under various conditions.
- They proved results for random geometric objects and central limit theorems in geometry. They showed the exact analogue of Jarnik's classical convex lattice polygon theorem when the norm is not Euclidean.
- They have proved an important result on the algorithmic version of the Local Lemma.
- They determined the (asymptotic) number of various mathematical objects in the Erdős-Kleitman-Rothschild Theory, in case of hereditary graph properties and for some kind of groups.
- They proved numerous results about dominating sets in infinite graphs.
- They showed several intersection theorems on the subsets of a finite set.
- They proved several important results in coding theory and search theory, for example they determined all the codes where all the distances between the codes are three.
- They found deep results on the typical and atypical incidence structures of parametrized families of curves.
- They also proved that both the Connes embedding conjecture and the Lueck-Sauer-Wegner generalized determinant conjecture hold for treeable group action.
- Results on pseudorandom number sequences were found and proved.
- Description of edge-critical hypergraphs was given for special hypergraph properties.

Department of Geometry

- They investigated the conjecture that among the graphs with given chromatic number, the complete graph has the smallest intersection number. They proved it for the case when the chromatic number is at most 17.
- They proved that for any set of even number of points in the plane one can construct a polygonal path through the points with bounded angles. For random points, a much stronger result was proved.
- They showed a lower bound for the size of a blocking set of a plane set of points.
- They investigated special types of chromatic numbers of a graph and proved positive and negative results about their relative sizes, estimating them by the functions of each other.
- They generalized the Ramsey type result of Erdős about empty convex polygons when the points are replaced by convex discs.

- They have investigated the mean width of the convex hull of n random points for more general convex bodies than so far. The asymptotic behaviour of the mean width difference was determined. Furthermore the order of the variance was determined, and the strong law of large numbers was verified for this problem.
- They have proved a better error terms for the stability versions of the classical Blaschke-Santalo and the affine isoperimetric inequalities.
- They obtained a lower estimate in n-space for the density of a ball lattice, supposing that it intersects any affine k-subspace.
- They proved that in at least three dimensional space, the infimum of the volumes of convex polytopes with any given facet areas is zero.

Department of Set Theory and General Topology

- They proved the consistency of the existence of a non-first countable compact space in which there is no convergent sequence whose length is the first uncountable ordinal. In addition to this, they gave numerous general results concerning the convergence and character spectra of compact spaces.
- It is known that a compact space is metrizable provided that such are all its subspaces whose size is the first uncountable cardinal. They showed that the same statement for locally compact spaces is already independent of the usual axioms of set theory. This answered a question that had been open for 15 years.
- They carried on the study of realization of cardinal sequences as the cardinal sequence of a compact scattered space.
- Using chains of elementary submodels they gave new proofs of the classical decomposition theorems of Nash-Williams.
- They achieved important results in a very fashionable topic which, among others, has been investigated by several Fields medalists. They proved that the measure of the intersection of fractal sets is very non-stable.
- They also managed to give a full description of the continuous rigid functions of two variables.
- They succeeded in proving Vaught's conjecture from 1961 for a new class of theories: if a first order theory T has uncountably many pairwise non-isomorphic countable models, then T has continuum many such models.
- According to a classical result of Robinson and Łoś, first order resolution endowed with paramodulation forms a refutation-complete calculus. They found a new, simple proof for this well known fact.

Department of Number Theory

- The study of the distribution of primes was continued. It was proved that assuming the Bombieri-Vinogradov theorem with any exponent exceeding 1/2 there is an integer d (for which an effective bound was given) such that there are arbitrarily long arithmetic progressions composed exclusively of primes p with the property that each $p+d$ is also prime, that is, there are arbitaritly long arithmetic progressions of generalized twin primes. Assuming the stronger Elliott-Halberstam conjecture this number d is an even integer between 2 and 16.
- Poisson's classical summation formula (which asserts that the sum of values of a real function at integer places is equal to the corresponding sum of its Fourier transform) was extended. In the new formula integers are replaced by a different discrete set, the set of eigenvalues of a hyperbolic Laplace operator, and Fourier transform is replaced by a recently discovered integral transform (Wilson transform). An important new

phenomenon is the appearance of triple product integrals, familiar from the theory of automorphic forms.
- A Burgess-type subconvex estimate was proved for twisted Hilbert modular L-functions which improves results of Cogdell-Piatetski-Shapiro-Sarnak and Venkatesh. As an application, this yields improved estimates for the number of representations by a positive definite ternary quadratic form over a completely real algebraic number field.
- The following problem was investigated. Given a set of integers in which either the number of sums or the number of differences is near to its possible maximal value, how far can the other quantity deviate from its maximum.

Department of Probability Theory and Statistics

- It was proven that the diffusion of two particles with mass transportation is a mixture of Brownian motions.
- Theorem of iterated logarithm and Erdős type of invariance theorems were proved for random walks on so called two-dimensional comb.
- Asymptotic behavior was established for truncated partial sum of independent random variables with infinite variance.
- A new improved proof was given for a theorem about polynomials of Gaussian variables.
- The concept of distance correlation was introduced and its relation with the so called Brownian motion type correlation was determined. An interesting property of the distance correlation is that it is equal to zero only for independent random variables in case of any distribution.
- An inequality about metric concentration was proven in case of Euclidean distance.
- The proportional and accelerated hazard rate models were unified and the novel model was applied to data of National Cancer Registry.

Applied research

The main research profile of the Rényi Institute is pure (theoretical) research and therefore most of the ongoing research projects do not have immediate applications. However, the already established applied research divisions, mostly related to the Discrete Mathematics Department and the Probability Theory & Statistics Department, namely the bioinformatics, database research, information theory and mathematical immunology research groups have continued their research and obtained significant results. Many members of the Discrete Mathematics (and other) Departments continued their research on large random networks, including the classification of atypical subgraphs of randomly evolving communication networks. This question was raised by an NKTH (National Office for Research and Technology) research and development project concluded in 2008.

The cryptography project applied for and was awarded by the “Momentum” program of the Hungarian Academy of Sciences. It gave a new zest to continue the earlier, naturally mostly applied cryptology research. With the help of the project and a reorganizing of human resources a new, 11 member research group has been put together in the second half of the year and already obtained results on secret sharing protocols, anonymous broadcasting protocols and fingerprint codes.

Bioinformatics research at the institute has been continued. The most active calendar year of the “Hungarian Bioinformatics” Transfer of Knowledge EU FP6 project of the institute was the year 2009 with several external (foreign) visits of length 2-4 months and 3 international research conferences during the summer about phylogenetics, protein structure prediction and comparative genomics (the latest one was the first conference in the region organized in the

series of the international RECOMB CG conferences). Matching the earlier research projects and fitting with the research directions of the visitors and conferences, the main research topics remained genome rearrangement, insertion-deletion process (statistical alignment), phylogenetic (evolutionary) trees and networks and the quest for new statistical methods. The success of the cooperation is well represented by the papers written jointly with the researchers visiting the institute.

III. Presentation of national and international relations

Relations to higher education institutions

The research fellows of the institute participated in the work of several universities (e.g. ELTE, BME, University of Debrecen, University of Szeged) as instructors, especially in giving courses for senior or PhD students of mathematics and physics. The joint mathematical PhD and MSc programs of the Central European University and the institute began its seventh year. The department has 21 PhD students and 12 MSc students. The professors of the program (the faculty of the Mathematics Department) are mostly the fellows of the institute, they gave 25 courses last year. Counting the students of all universities, the researchers of the institute supervised 29 PhD students, 18 MSc students and 2 scientific scholarship holders. The research fellows of the institute provide the majority of the professors for the Budapest Semesters in Mathematics program – organized mostly for American students. Thirty-nine fellows of the institute (58 pc. of them) instructed in some Hungarian university. The number of classes they instructed exceeded 3900 in 2009.

Keeping with the tradition, the institute hosted four university professors as visiting scholars in 2009, altogether for 15 months. During their selection it was very important that their subject was closely related to the research profile of the institute. Unfortunately, due to the financial difficulties, the institute is forced to decrease this activity in the future, unless separate sources will be provided for this priority goal of the Academy.

National relations

The weekly research seminars of the institute continued. The importance of the seminars exceeds the framework of the institute. They influence the trends in all of Hungarian mathematical research.

The fellows of the institute participate in the mathematical public life well above their number share. They have important positions in the Search Committee of the Academy, the Mathematics Department of the Academy and its committees, the Council of the Research Institutes of the Academy, the Curatory of Sciences, the committees of OTKA, the Hungarian Accreditation Committee, and the Board of the János Bolyai Mathematical Society. The president of the János Bolyai Mathematical Society, the chairman of the Bolyai Research Fellowship Curatory, the mathematics coordinator of the Board of Young Researchers of HAS, the vice chairman of the Department of Mathematics of HAS, the chairman and the secretary of the Mathematics Committee of HAS, the chairman and vice-chairman of the Mathematics Doctoral Committee (HAS), the chairman and vice-chairman of the CEU Mathematics Doctoral Program, the chairman of Mathematics Panel of OTKA and the chairman of the Hungarian Accreditation Committee, Mathematics Education Committee are fellows of the institute.

International relations

Traditionally, the researchers of the institute have very wide-range international relations. The cooperation manifests mostly in joint publications, study trips in both directions, joint projects and jointly organized conferences. The fellows of the institute participated in the organization of 29 conferences and workshops in 2009. Twelve of them took place in the institute itself. In general, these cooperations do not need institutional form, but their success is indicated by the large number of joint publications.

The scientific visits in the framework of both the bilateral agreements of the academy and the contracts of TéT successfully helped the scientific cooperation. They opened the possibility for successful joint research, useful exchange of information and sometimes for participation in conferences.

The fellows of the institute possessed ten memberships in international scientific committees and 78 memberships in editorial boards of international journals. They gave 233 lectures at international conferences, many of which were invited or plenary talks.

The institute hosted 16 visiting scholars in two EU projects, altogether for 33 months. At the same time, visitors came from non-European and European countries with the support from various sources like Fulbright scholarships, TéT contracts, OTKA projects, bilateral agreements of the Academy and independent sources provided by the visitors. Altogether, the institute hosted more than 50 visitors in 2009, not counting conference participants.

The scientific meetings organized completely or partially by the institute were as follows.

- Workshop on Combinatorial Geometry, February 6, 2009
- Paul Turán Memorial Lectures, February 17–19, 2009
- Lecture Series on Epstein-Barr Virus, April 21–27, 2009
- 6th Japanese-Hungarian Symposium on Discrete Mathematics and Its Applications, May 16–19, 2009
- Surface Singularities and Related Topics, June 6–7, 2009
- Algorithmic and Combinatorial Geometry, June 15–19, 2009
- Phylogenetics 2009, June 22–25, 2009
- 8th Summer School in Potential Theory, July 6–12, 2009
- Protein Structure Prediction Workshop, August 26–29, 2009
- 3rd Workshop in Fourier Analysis and Related Fields, September 17–23, 2009
- Seventh Annual RECOMB Satellite Workshop on Comparative Genomics, September 27–29, 2009
- Asymptotic Results in Probability and Statistics, November 5–7, 2009

IV. Brief evaluation of successful national and international grants

National grants

Despite the fact that the research groups of the institute have been as successful as in previous years with the Hungarian OTKA (Hungarian Scientific Research Fund) project proposals, the total income of the institute from these national projects was lower than the previous two years. This was due mainly to the international economic crisis and therefore the narrowing number of accepted projects and financial resources distributed by them. Still, it is safe to say that all faculty of the institute are members of at least one running OTKA project and that by far the institute submits the highest number of OTKA project proposals in mathematics.

Apart from the OTKA projects, there have been no other significant project proposal calls in 2009 for theoretical research in mathematics. The institute had two project proposal plans for the fall round of the TECH09 call of NKTH (National Office for Research and Technology), but the call was delayed until 2010.

Therefore the "Momentum" call for special projects of the Hungarian Academy of Sciences and the submission and founding of the cryptography project was especially important for the institute, since, though is definitely an applied mathematics project, it does not carry the urgent needs of reaching and demonstrating concrete applied results immediately and the 3+2 years length of the project is more appropriate to the usual length of mathematical investigations than the typical 1-3 years length of the other research and development projects.

International grants

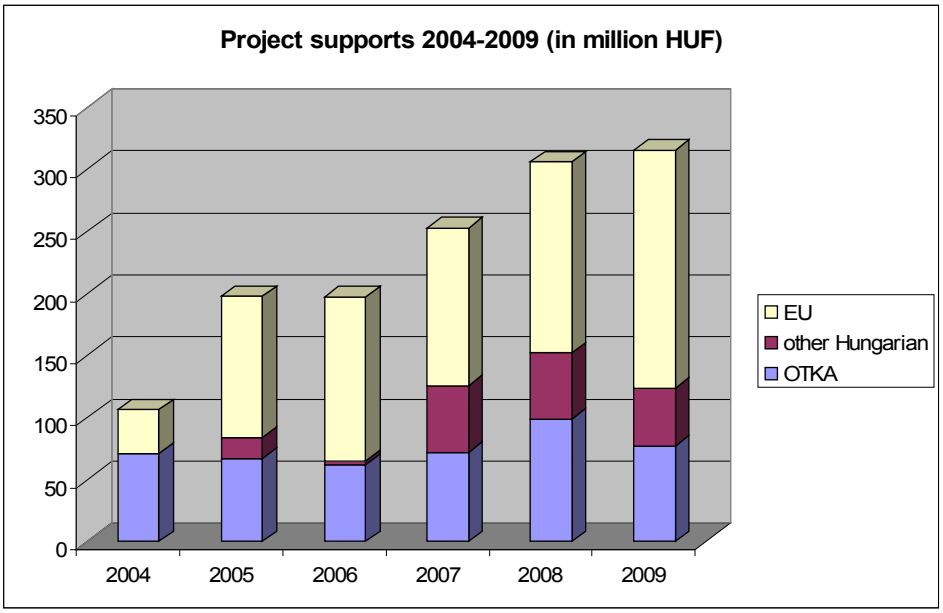
The majority of the running international research projects of the institute were still the mobility type Transfer of Knowledge projects of FP6 from 2004 and 2005. The foreign researchers staying at the institute for a longer period successfully joined the research topics, opened new international research connections with their series of lectures and personal consultations. Unfortunately all but one of these projects already ended in 2009, the last will end in September 2010 and the new FP7 program of the EU does not contain a similar project call. Nevertheless, due to the delayed financing of the finished or nearly-finished projects, they significantly contributed to the finances of the institute in 2009.

Recently a new type of research projects of the European Research Council, namely the so called Starting Independent Researcher and Advanced Investigators Grants were introduced. These projects basically funds with even several million euros the research of small research groups run by the principal investigator (the starting independent researcher or the advanced investigator). There are a low number of funded projects and therefore they are highly competitive research calls. Therefore, though the researchers of the institute are very active on these calls (with 2 to 5 project applications per call), there was no success in 2009 similar to the 2008 one, when the research project "Gaps between primes and almost primes. Pattern in primes and almost primes. Approximations to the twin prime and Goldbach conjecture" of a team lead by a professor of the institute and consisting of several other internationally renowned members of the institute was awarded l.376.400 euros for a one year research period. Still, it is considered to be a great success that the proposal of a senior researcher of the institute, has reached the second round of the Advanced Investigators Grants call.

Of smaller scale, but still important project opportunities are the individual PEOPLE projects of the FP7. Four researchers began their research projects during 2009 at the Rényi Institute either in the frame of a longer (1 or 2 years long) research visit or – in case of the younger generation Hungarian researchers – as the first step of their reintegration into Hungary.

Altogether, though we have not witnessed the earlier dynamic increase in the income of Hungarian and international research projects (the income of the Hungarian projects even decreased), the total income of the projects slightly exceeded that in the year 2008 and reached 70% of the subsidy given by the state budget and 40% of the total income of the institute. This happened despite the narrowing number of possibilities, but due to the still-increasing incomes of the international projects and the "Momentum" project of the HAS. Because the immediate support from the State (via the Academy) contributes in a lesser and lesser extent to the successful function of the institute, and in 2009 it decreased drastically, the institute depends more and more on the income of the projects. Still, it should be noted that considering the recent trends of Hungarian and international project proposals, the tendency of increasing income will not be continued in 2010, rather income will be significantly lower.

The diagram below shows the receipts of the projects during the last 6 years.



V. The most important publications and patents in 2009

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2. Ambrus G, Bárány I: Longest convex chains, Random Structures & Algorithms 35(2): 137-162 (2009)
3. Berkes I, Weber M: On the convergence of $\sum c_k f(n_k x)$, Memoirs of the American Mathematical Society 201(943): 1-72 (2009)
4. Csáki E, Csörgő M, Földes A, Révész P: Random walk local time approximated by a Brownian sheet combined with an independent Brownian motion, Annales de l'Institut Henri Poincaré Probabilités et Statistiques 45(2): 515-544 (2009)
5. Kim H, Toroczkai Z, Erdős PL, Miklós I, Székely LA: Degree-based graph construction, Journal of Physics A-Mathematical and Theoretical 42(39): 392001 (2009)
6. Benko D, Kroó A: A Weierstrass-type theorem for homogeneous polynomials, Transactions of the American Mathematical Society 361(3): 1645-1665 (2009)
7. Jaming P, Matolcsi M, Révész SzGy: On the extremal rays of the cone of positive, positive definite functions, Journal of Fourier Analysis and Applications 15(4): 561-582 (2009)
8. Csűrös M, Miklós I: Streamlining and large ancestral genomes in Archaea inferred with a phylogenetic birth-and-death model, Molecular Biology and Evolution 26(9): 2087-2095 (2009)
9. Némethi A, Okuma T: On the Casson invariant conjecture of Neumann-Wahl, Journal of Algebraic Geometry 18(1): 135-149 (2009)
10. Petz D, Szántó A, Weiner M: Complementarity and the algebraic structure of four-level quantum systems, Infinite Dimensional Analysis Quantum Probability and Related Topics 12(1): 99-116 (2009)
11. Goldston DA, Pintz J, Yildrim CY: Primes in tuples I, Annals of Mathematics 170(2): 819-862 (2009)
12. Bonami A, Révész SzGy: Integral concentration of idempotent trigonometric polynomials with gaps, American Journal of Mathematics 131(4): 1065-1108 (2009)
13. Ruzsa IZ: Sumsets and entropy, Random Structures & Algorithms 34(1): 1-10 (2009)
14. Balogh J, Bollobás B, Simonovits M: The typical structure of graphs without given excluded subgraphs, Random Structures & Algorithms 34(3): 305-318 (2009)
15. Simonyi G, Tardos G, Vrecica ST: Local chromatic number and distinguishing the strength of topological obsructions, Transactions of the American Mathematical Society 361(2): 889-908 (2009)
16. Lisca P, Ozsváth P, Stipsicz AI, Szabó Z: Heegard Floer invariants of Legendarian knots in contact three-manifolds, Journal of the European Mathematical Society 11(6): 1307-1363 (2009)
17. Harari D, Szamuely T: Galois sections for abelianized fundamental groups, Mathematische Annalen 344(4): 779-800 (2009)